Partitioned manifolds and invariants in dimensions 2, 3, and 4

Booss-Bavnbek, Bernhelm; Wojciechowski, K.P.

Publication date:
1993

Document Version
Publisher's PDF, also known as Version of record

Citation for published version (APA):
Booss-Bavnbek, B., & Wojciechowski, K. P. (1993). *Partitioned manifolds and invariants in dimensions 2, 3, and 4*. Roskilde Universitet. Tekster fra IMFUFA No. 260 http://milne.ruc.dk/ImfufaTekster/

TEKST NR 260 1993

B. Booß-Bavnbek
K. P. Wojciechowski

PARTITIONED MANIFOLDS AND INVARIANTS IN DIMENSIONS 2, 3, AND 4

TEKSTER fra
IMFUFA
ROSKILDE UNIVERSITETSCENTER
INSTITUT FOR STUDIET AF MATEMATIK OG FYSIK SAMT DERES FUNKTIONER I UNDERVISNING, FORSKNING OG ANVENDELSER

IMFUFA, Roskilde Universitetscenter, Postboks 260, 4000 Roskilde

PARTITIONED MANIFOLDS AND INVARIANTS IN DIMENSIONS 2, 3, AND 4

by: B. Booß-Bavnbek and K. P. Wojciechowski

IMFUFA tekst nr. 260/93 8 pages ISSN 0106-6242

Abstract.

Emphasizing special features of manifolds with symmetry in dimensions 2, 3, and 4 we develop various simple approaches to index theory over partitioned manifolds.

PARTITIONED MANIFOLDS AND INVARIANTS IN DIMENSIONS 2, 3, AND 4

B. BOOSS-BAVNBEK
Institut for matematik og fysik
Roskilde University
DK-4000 Roskilde, Denmark
e-mail: booss@jane.ruc.dk

and

K. P. WOJCIECHOWSKI*
Dept. Math., IUPUI
Indianapolis IN 46202, U.S.A.
e-mail: kwojciec@indyvax.iupui.edu

Abstract. Emphasizing special features of manifolds with symmetry in dimensions 2, 3, and 4 we develop various simple approaches to index theory over partitioned manifolds.

Key words: Dirac operators, elliptic boundary problems, index theory, partitioned manifolds, spectral flow, surgery

Eugene Wigner's sarcastic remark in [1], that *mathematics is the science of skillful operations with concepts and rules invented just for this purpose*, might have been read by generations of mathematicians as a blank cheque to work with concepts *not chosen for their conceptual simplicity... but for their amenability to clever manipulations and to striking, brilliant arguments.* Since then it became socially accepted among mathematicians not to keep track of what was going on in physics. For a while, algebraic topology became more fashionable than partial differential equations.

Surprisingly, Wigner's expectation of *the mathematical concepts cropping up in physics* also held for the most elaborate and remote concepts of modern differential topology. To judge from the terminology or the content of the papers, some issues of learned journals in differential geometry and in mathematical physics can hardly be distinguished from each other. Mathematicians tend to give that fact an emphatic interpretation like Michael Atiyah [2]: *When the excitement is over and a proper perspective can be achieved the present decade* (written in 1989) *may well stand out as a landmark comparable to that of Einstein's Theory.* Physicists, trained in Wigner's way of thinking, do not always evaluate so positively the inflation of new topological and differential geometrical concepts turning up especially in quantum field theory and elementary particle theory. Usually they have two objections: They ask for the phenomena, which according to Wigner's first quality assessment criterion should find a *close and accurate description* by help of the fancy mathematical concepts. This question we leave to the physicists. And they ask for the *ingenious*

* Work supported in part by NSF grant no. DMS-9105057.

logical operations which appeal to our aesthetic sense both as operations and also in their results of great generality and simplicity, Wigner's second quality assessment criterion. Is not the terminology becoming too much mathematized, too esoteric and too bizarre?

To this question we have an answer regarding the global analysis of elliptic boundary value problems: No! Understanding internal symmetries of partial (especially elliptic) differential equations in terms of invariants looks like a puzzle. But it is not since the basic building blocks in analysis and topology are the same: manifolds with boundary. Of course, the emphasis is different. In the *theory of partial differential equations* one traditionally treats higher order differential equations of greatest generality on bounded planar regions or open regions in higher dimensional Euclidean space, subject to local Dirichlet/Neumann or very abstract pseudo-differential boundary conditions. In *topology* one investigates combinatorial properties of quite general manifolds made up by glueing of simple pieces, celles or singular simplices. A link between these two very different theories is provided by *differential topology* which treats mainly geometrically defined first order differential operators of Dirac type, acting on sections of bundles of Clifford modules, subject to strong symmetry principles and concrete global spectral boundary conditions. That provides an abundance of cross references between analysis and topology. Whether they have a deep physical meaning or not, these cross references are interesting mathematics.

Computations of invariants like the index, the eta-invariant, and the determinant for a Dirac operator over a closed manifold can be in some cases reduced to the following scheme: We divide the manifold into two parts. One part may be complicated from a topological point of view; but the analysis is easy on this part. The other part may be topologically trivial, but interesting from the analytical point of view. Here we make explicit computations. This is the idea e.g. behind the recent work of Kirk and Klassen [3], [4] on Casson's invariant which gives one reason, why it is important to study the decomposition of manifolds and operators into problems over manifolds with boundary.

Let us look at a few concrete calculations to illustrate the most basic concepts and to argue for our philosophy that manifolds with boundary are, in principle, easier to grasp than closed manifolds. We shall restrict ourselves to the easiest case of the index. Modifications of our approach for the slightly more intricate situation of the eta-invariant and the truly more intricate situation of the determinant and torsion will be worked out separately.

1. A Simple Proof of the Index Theorem on S^2

Consider an elliptic differential or pseudo-differential operator over a closed manifold (i.e. compact and without boundary). The index denotes the difference of the dimensions of the kernel and the cokernel of the operator. It is continuous on a suitable space of Fredholm operators and hence locally constant on the connected components. Then the Atiyah-Singer index theorem expresses the index in terms of topological characteristics of the operator's coefficients.

We consider the simplest possible two-dimensional case and write the 2-sphere

S^2 as the closed double of the 2-disc $D^2 := \{(x,y) \mid x^2+y^2 \le 1\}$. We show that, correspondingly, up to homotopy any elliptic operator on S^2 is completely determined by the pasting of trivial pieces of the operator. More precisely, we construct two operators A and A' which are, in topological language, non-trivial, i.e. their principal symbols define the two generators of the K-group $K(TS^2)$, one with vanishing index, the other with non-vanishing index. The building blocks are provided by the Cauchy-Riemann operator $\bar\partial : C^\infty(D^2) \to C^\infty(D^2)$, where $\bar\partial := \frac{1}{2}(\partial/\partial x + i\partial/\partial y)$. In polar coordinates out of the origin, this operator has the form $\frac{1}{2}e^{i\varphi}(\partial_r + (i/r)\partial_\varphi)$. Therefore, after some small smooth perturbations (and modulo the factor $\frac{1}{2}$), we assume that $\bar\partial = e^{i\varphi}(\partial_r + i\partial_\varphi)$ in a certain collar neighbourhood N of the boundary. Then $(\bar\partial)^* = e^{-i\varphi}(-\partial_u + i\partial_\varphi + 1)$ $(u = r-1)$ in N, and

$$A = \bar\partial \cup (\bar\partial)^* : C^\infty(S^2; H^1) \to C^\infty(S^2; H^{-1}) \tag{1}$$

is a well-defined elliptic differential operator. Here for integer k, H^k denotes the Hopf bundle, which is obtained from two copies of $D^2 \times \mathbf{C}$ by the identification $(z,w) = (z, z^k w)$ near the equator. By definition, the kernels of A and its adjoint operator are isomorphic, hence $\operatorname{index} A = 0$.

To produce an operator with a nontrivial index, we define an operator $\bar\partial_1$ on the upper hemisphere, equal to $\bar\partial$ outside of N. On N we define:

$$\bar\partial_1 := e^{i\varphi}(\partial_u + i\partial_\varphi - \chi(u)), \tag{2}$$

where $\chi(u) \ge 0$ is a smoothing function equal 1 close to the equator $u = 0$. Then the operator

$$A' := \bar\partial_1 \cup (\bar\partial)^* : C^\infty(S^2; H^2) \to C^\infty(S^2; H^0)$$

is well defined. The only difference from the operator $\bar\partial \cup (\bar\partial)^*$ is the modification on the collar N.

To compute $\operatorname{index} A' - \operatorname{index} A$ observe that the index of any Dirac operator over a closed manifold M is given by a local formula

$$\operatorname{index} A = \int_M \alpha(x),$$

where $\alpha(x)$ is a density given at the point x by a complicated algebraic formula in terms of coefficients of the operator A in x (accounted for e.g. in [5]). It is a consequence of the locality of the index expressed in the preceding formula that we get:

$$\operatorname{index} A' = \operatorname{index} A' - \operatorname{index} A = \operatorname{index}[e^{i\varphi}(\partial_u + i\partial_\varphi - \chi(u))], \tag{3}$$

where the operator on the right side is a well-defined elliptic operator on the torus $S^1 \times S^1$. Its index coincides with the index of the elliptic operator $T := \partial_u + i\partial_\varphi - u$ acting on functions $f(u,\varphi)$ from $C^\infty(\mathbf{R} \times \mathbf{S}^1)$ which satisfy the periodicity condition:

$$f(u+1,\varphi) = e^{-i\varphi} f(u,\varphi). \tag{4}$$

Separation of variables and series expansion of f shows that there is no non-trivial solution of $Tf = 0$ with the required periodicity and that the kernel of T^* with the required periodicity is one-dimensional, hence

$$\operatorname{index} A' = -1. \tag{5}$$

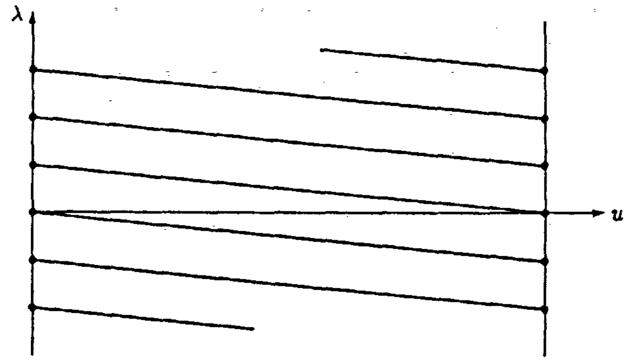

Fig. 1. The spectrum of $\{i\frac{d}{d\varphi} - u\}_{u\in I}$

More generally, this argument proves the following result:

Theorem 1 *Let A'^k denote an operator over S^2 which is equal to $\bar\partial$ on $D^2 \setminus N$, $(\bar\partial)^*$ on the second copy of D^2, and $e^{i\varphi}(\partial_u + i\partial_\varphi - k\chi(u))$ on N. The operator A'^k is in fact the operator $A \otimes Id_{H^k} : C^\infty(S^2; H^{k+1}) \longrightarrow C^\infty(S^2; H^{k-1})$ and we have the following index theorem:*

$$\operatorname{index} A'^k = -k\,. \tag{6}$$

Remarks 2 (a) Alternatively, we can prove (5) by a spectral argument. Consider the family $\{B_u = i\partial_\varphi - u\}$ of self-adjoint differential operators over the circle S^1, parametrized by $u \in S^1 = I/\{0,1\}$. Such families have only one topological invariant, the *spectral flow* $\mathrm{sf}\{B_u\}$. It is just the difference between the number of eigenvalues of B_u, which change the sign from $-$ to $+$ as u goes from 0 to 1, and the number of eigenvalues, which change the sign from $+$ to $-$. With some work, involving K-theory, one finds some very fundamental relations between *families of self-adjoint elliptic differential operators* $\{B_u\}$ over a closed manifold M parametrized by S^1; their *suspension* $\partial_u + B_u$ which is an elliptic differential operator over $M \times S^1$; and their *desuspension* $\mathrm{P}_\geq - \Psi\mathrm{P}_<$ which is an elliptic pseudo-differential operator over M. Here $\mathrm{P}_\geq$ and $\mathrm{P}_<$ denote the spectral projections of the operator B_0 on the eigenspaces corresponding to the non-negative part and the negative part of the spectrum; and Ψ is a unitary morphism chosen in such a way that it provides a unitary equivalence $B_1 = \Psi^{-1}B_0\Psi$, here $\Psi(\varphi) = e^{i\varphi}$. One gets for $T = \partial_u + B_u$

$$\operatorname{index}(\partial_u + B_u) = \mathrm{sf}\{B_u\} = \operatorname{index}(\mathrm{P}_\geq - \Psi\mathrm{P}_<), \tag{7}$$

hence, especially, $\operatorname{index} A' = \mathrm{sf}\{B_u\} = -1$, see Figure 1.

(b) Direct generalization yields the corresponding index theorem for Dirac operators on even-dimensional spheres which gives the Atiyah-Singer index theorem on these spheres by stable homotopy. That result, together with the computation of the signature operator on complex projective spaces, was the basis of the first proof of the index theorem.

(c) Notice that the cutting and pasting procedure explained above for the 2-sphere also provides a direct proof of the Atiyah-Singer index theorem for elliptic pseudo-differential operators on arbitrary closed Riemann surfaces.

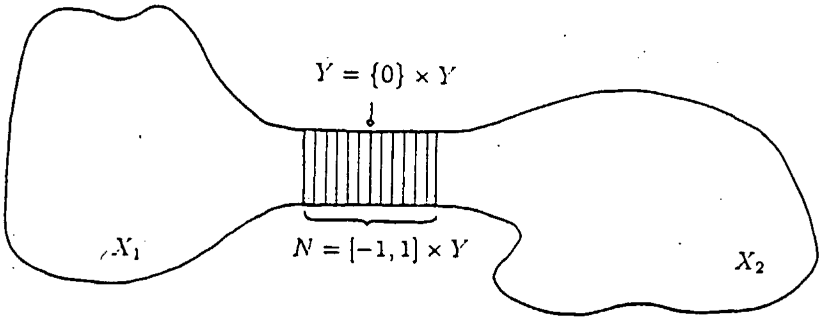

Fig. 2. The partitioned manifold $M = X_1 \cup X_2$

2. Index Theory on Partitioned Manifolds

Let $M = X_1 \cup X_2$ be an n-dimensional closed partitioned manifold with $\partial X_1 = \partial X_2 = X_1 \cap X_2 = Y$, let S be a $C\ell(M)$-module, and let $A : C^\infty(M;S) \longrightarrow C^\infty(M;S)$ be a *generalized Dirac operator*, i.e. a linear differential operator of first order which can be written in terms of a local orthonormal frame $v_1, \ldots, v_n$ of TM as

$$As|_x = \sum_{\nu=1}^{n} v_\nu \cdot (D_{v_\nu} s)|_x ,$$

where $\cdot$ denotes Clifford multiplication. We assume n even and get a *chiral decomposition*

$$A = \begin{pmatrix} 0 & A^- \\ A^+ & 0 \end{pmatrix} .$$

The *partial Dirac operators* $A^\pm$ are especially interesting in index theory since they are elliptic, but in general not self-adjoint and provide interesting integer-valued invariants as their indices. Like the Cauchy-Riemann operator all partial Dirac operators A^+ can be written in product form

$$A^+ = G(u,y)(\partial_u + B_u) \tag{8}$$

in a bicollar neighbourhood N of Y, where u denotes the normal coordinate running from X_1 to X_2, see Figure 2. We assume that the metrics of the manifold and the Clifford module bundle are product close to Y, hence G and B_u independent of u. Notice that $G(y)$ is Clifford multiplication with the inward oriented normal tangent vector and B_0 is again a Dirac operator, sometimes called the *tangential operator* or the *Hamiltonian*. Due to (8) all the constructions shown for the two-sphere and the Cauchy-Riemann operator have straight forward generalizations.

Let Φ be a unitary automorphism of $S^+|_Y$ (possibly with a shift f in the basis Y). Let a denote the principal symbol of the operator A^+. In a bicollar neighbourhood of Y, the symbol a has the form:

$$a(u,y;\nu,\zeta) = G(y)(i\nu + b(y;\zeta)) .$$

We assume *consistency* of Φ and G, namely that for all $y \in Y$ and $\zeta \in T^*Y_y$:

$$\Phi(y)G(y) = G(f(y))\Phi(y) \quad \text{and} \quad \Phi(y)b(y;\zeta) = b\left(f(y);(f^{-1})^*(\zeta)\right)\Phi(y) .$$

Theorem 3 ([6]) *Glueing $A_1 := A^+|X_1$ with $A_2 := A^+|X_2$ in a bicollar neighbourhood of Y by consistent Φ defines a new Dirac operator A^Φ over M with*

$$\operatorname{index} A^\Phi - \operatorname{index} A^+ = \operatorname{index} T^\Phi = \operatorname{sf}\{B_u\} = \operatorname{index}(\mathrm{P}_< - \Phi \mathrm{P}_\geq).$$

Here $\{B_u\}$ is a family connecting B_0 and $\Phi^{-1}B_0\Phi$; T^Φ denotes the elliptic differential operator given by the formula $G(\partial_u + B_u)$ over the mapping torus $(I \times Y)/f$; and $\mathrm{P}_\geq$, $\mathrm{P}_<$ denote the spectral projections of B_0.

Corollary 4 *The index of the corresponding linear conjugation problem on the manifold M equals the index of the operator A^Φ.*

3. Additivity Formula for Problems of Atiyah-Patodi-Singer Type

The principal symbol p_+ of the spectral projection $\mathrm{P}_\geq$ is the projection onto the eigenspaces of the principal symbol $b(y,\zeta)$ of B_0 corresponding to non-negative eigenvalues. We call the space of pseudo-differential projections with the same principal symbol p_+ the *Grassmannian* Gr_{p_+}. It has enumerable many connected components; two projections P_1, P_2 belong to the same component, if and only if the *virtual codimension*

$$\mathrm{i}(P_2, P_1) := \operatorname{index}\{P_2P_1 : \operatorname{range}P_1 \longrightarrow \operatorname{range}P_2\} \tag{9}$$

of P_2 in P_1 vanishes; the higher homotopy groups of each connected component are given by Bott periodicity. The Grassmannian is a natural space for global elliptic boundary value problems, see [6] for the mathematics and [7] for the physics.

More precisely: The L^2 realization $(A_1)_{P_1}$ which acts like A_1 and is determined by

$$\operatorname{dom}(A_1)_{P_1} := \{s \in \mathcal{H}^1(X_1; S^+) \mid P_1(s|_Y) = 0\},$$

is a Fredholm operator from $L^2(X_1; S^+)$ to $L^2(X_1; S^-)$. Its index is determined by a generalization of the *Atiyah-Patodi-Singer index formula*

$$\operatorname{index}\,(A_1)_{P_1} = \int_{X_1} \alpha(x) - \tfrac{1}{2}\,(\eta_B(0) + \dim\ker B) + \mathrm{i}(P_1, \mathrm{P}_\geq). \tag{10}$$

Here $\alpha(x)$ denotes the index density of A_1 accounted for above and

$$\eta_B(z) := \sum_{\lambda \in \operatorname{spec} B \setminus \{0\}} \operatorname{sign}\lambda |\lambda|^{-z} \tag{11}$$

denotes the *η-function* of A_1's tangential part B. It is (i) well defined through absolute convergence for $\Re(z)$ large; (ii) it extends to a meromorphic function in the complex plane with isolated simple poles; (iii) its residues are given by a local formula; and (iv) it has a finite value at $z = 0$ (see e.g. [5]).

Let $A^+|_N = G(\partial_u + B)$, i.e. independent of u. This can be obtained when the metrics of the manifold and the Clifford module are product close to Y. Then

$$A_1|_{[-1,0]\times Y} = (-G)(-\partial_u - B) \quad \text{and} \quad A_2|_{[0,1]\times Y} = G(\partial_u + B), \tag{12}$$

and it follows that $(p_1)_+ = \mathrm{id} - (p_2)_+$.

Theorem 5 *Let P_i be projections belonging to $\mathrm{Gr}_{(p_i)+}$, $i = 1, 2$. Then*

$$\operatorname{index} A^+ = \operatorname{index}(A_1)_{P_1} + \operatorname{index}(A_2)_{P_2} - \mathbf{i}(P_2, \mathrm{Id} - P_1). \tag{13}$$

Note. It is immediate that $\mathbf{i}(P_2, \mathrm{Id} - P_1) = \operatorname{index}(G(\partial_u + B); P_2, P_1)$ where the last operator is on $[0,1] \times Y$ with boundary condition P_2 at $u = 0$ and P_1 at $u = 1$. Formulas similar to (13) hold also for some other interesting analytical invariants like the eta-invariant of Dirac operators on odd-dimensional manifolds and the spectral flow of families of operators (see [8], [9], and [10]).

Proof From (10) we get

$$\operatorname{index}(A_1)_{P_1} = \int_{X_1} \alpha_{A_1}(x) - \tfrac{1}{2}(\eta_{-B}(0) + \dim\ker(-B)) + \mathbf{i}(P_1, \mathrm{P}_{\geq}(-B))$$

and

$$\operatorname{index}(A_2)_{P_2} = \int_{X_2} \alpha_{A_2}(x) - \tfrac{1}{2}(\eta_B(0) + \dim\ker B) + \mathbf{i}(P_2, \mathrm{P}_{\geq}(B)).$$

Here $\alpha_{A_i} := \alpha|_{X_i}$. Then

$$\begin{aligned} \operatorname{index}(A_1)_{P_1} &+ \operatorname{index}(A_2)_{P_2} \\ &= \int_M \alpha(x) - \dim\ker B + \mathbf{i}(P_1, \mathrm{P}_{\geq}(-B)) + \mathbf{i}(P_2, \mathrm{P}_{\geq}(B)). \end{aligned} \tag{14}$$

Since

$$\begin{aligned} \mathbf{i}(P_1, \mathrm{P}_{\geq}(-B)) &= -\mathbf{i}(\mathrm{Id} - P_1, \mathrm{Id} - \mathrm{P}_{\geq}(-B)) \\ &= \mathbf{i}(\mathrm{Id} - \mathrm{P}_{\geq}(-B), \mathrm{Id} - P_1) = \mathbf{i}(\mathrm{P}_{>}(B), \mathrm{Id} - P_1) \end{aligned}$$

and $\mathbf{i}(\mathrm{P}_{\geq}(B), \mathrm{P}_{>}(B)) = -\dim\ker B$ we get

$$\begin{aligned} \mathbf{i}(P_1, \mathrm{P}_{\geq}(-B)) &= \mathbf{i}(\mathrm{P}_{\geq}(B), \mathrm{P}_{>}(B)) + \mathbf{i}(\mathrm{P}_{>}(B), \mathrm{Id} - P_1) - \mathbf{i}(\mathrm{P}_{\geq}(B), \mathrm{P}_{>}(B)) \\ &= \mathbf{i}(\mathrm{P}_{\geq}(B), \mathrm{Id} - P_1) + \dim\ker B. \end{aligned}$$

Inserting the preceding result in (14) yields

$$\begin{aligned} &\operatorname{index}(A_1)_{P_1} + \operatorname{index}(A_2)_{P_2} \\ &\quad = \operatorname{index} A^+ - \dim\ker B + \mathbf{i}(P_2, \mathrm{P}_{\geq}(B)) + \mathbf{i}(\mathrm{P}_{\geq}(B), \mathrm{Id} - P_1) + \dim\ker B \\ &\quad = \operatorname{index} A^+ + \mathbf{i}(P_2, \mathrm{Id} - P_1). \end{aligned}$$

□

4. Specific Features in Dimensions 2, 3, and 4

Remarks 6 (a) Cutting and pasting of one single operator can generate (modulo stable homotopy) the whole space of elliptic operators over a partitioned closed manifold as was the case with the Cauchy-Riemann operator over S^2 in Section

1. This requires that the group of unitary consistent automorphisms is sufficiently rich. In other cases the group is too small to influence the index. This is the case for the signature and the Euler characteristic which remain invariant under cutting and pasting, i.e. they are *rigid* and behave additively.
(b) For each Dirac operator A over a compact manifold X with smooth boundary Y one has a second canonical element of Gr_{p_+} besides the spectral projection $\mathrm{P}_{\geq}$, namely the Calderón projection $\mathcal{P}_+$ which maps sections over the boundary onto the Cauchy data, i.e. the traces on the boundary of the solutions. By definition index $A_{\mathcal{P}_+}$ always vanishes.

Clearly, in dimension 2 the two projections for the Cauchy-Riemann operator coincide. In dimension 3 we consider a smooth flat connection on $X \times \mathrm{SU}(2)$ and the corresponding twisted signature operator over X with coefficients in $X \times \mathrm{su}(2)$ where su(2) denotes the Lie algebra of SU(2) (see [6]). The index of the corresponding Atiyah-Patodi-Singer problem does not vanish: it does not depend of the connection and can be expressed by $3 - 3g$ where $g \geq 2$ denotes the genus of Y. Exploiting an argument of [11], one can find an example in dimension 3 (with the genus of the connected boundary $g > 3$) where $\mathrm{P}_{\geq}$ and $\mathcal{P}_+$ change their connected components in Gr_{p_+} independently under a smooth change of the metric. In dimension 4 [12] shows that for the Euclidean Dirac operator on the 4-ball 'the boundary traces of zero mode spinors of even chirality coincide with the space of eigenfunctions of the Hamiltonian on the 3-sphere corresponding to non-negative eigenvalues', or in our language: the spectral projection and the Calderón projection coincide.

References

1. Wigner, E.: 1960, 'The unreasonable effectiveness of mathematics in the natural sciences', *Comm. Pure Appl. Math.* **13**, 1-14. Reprinted in *Symmetries and Reflections. Scientific Essays of Eugene P. Wigner*, Indiana Univ. Press, Bloomington, 1967, pp. 222-237.
2. Atiyah, M.F.: 1989, 'The frontier between geometry and physics', *Jber. d. Dt. Math.-Verein* **91**, 149-158.
3. Kirk, P.A., and Klassen, E.P.: 1993, 'Chern-Simons invariants of 3-manifolds decomposed along tori and the circle bundle over the representation space of T^2', *Comm. Math. Phys.* **153**, 521-557.
4. —, —: 1993, *Computing Spectral Flow via Cup Products*, Preprint, Indiana University, Bloomington.
5. Gilkey, P.B.: 1984, *Invariance Theory, the Heat Equation, and the Atiyah-Singer Index Theorem*, Publish or Perish, Wilmington. New revised edition in preparation.
6. Booß-Bavnbek, B., and Wojciechowski, K.P.: 1993, *Elliptic Boundary Problems for Dirac Operators*, Birkhäuser, Boston.
7. Witten, E.: 1988, 'Quantum field theory, Grassmannians, and algebraic curves', *Comm. Math. Phys.* **113**, 529-600.
8. Bunke, U.: 1993, *A Glueing Formula for the Eta-Invariant*, Preprint, Humboldt Universität, Berlin.
9. Wojciechowski, K.P.: 1993, *On the Additivity of the Eta-Invariant*, Preprint, IUPUI, Indianapolis.
10. Nicolaescu, L.: 1993, *The Maslov Index, the Spectral Flow, and Splittings of Manifolds*, Preprint, Michigan State University, East Lansing.
11. Hitchin, N.: 1974, 'Harmonic spinors', *Adv. Math.* **14**, 1-55.
12. Kori, T.: 1993, *Index of the Dirac Operator on S^4 and the Infinite Dimensional Grassmannian on S^3*, Preprint, Sci. and Eng. Res. Lab., Waseda University, Tokyo.

Liste over tidligere udkomne tekster tilsendes gerne. Henvendelse herom kan ske til IMFUFA's sekretariat tlf. 46 75 77 11 lokal 2263

217/92 "Two papers on APPLICATIONS AND MODELLING IN THE MATHEMATICS CURRICULUM"
by: Mogens Niss

218/92 "A Three-Square Theorem"
by: Lars Kadison

219/92 "RUPNOK - stationær strømning i elastiske rør"
af: Anja Boisen, Karen Birkelund, Mette Olufsen
Vejleder: Jesper Larsen

220/92 "Automatisk diagnosticering i digitale kredsløb"
af: Bjørn Christensen, Ole Møller Nielsen
Vejleder: Stig Andur Pedersen

221/92 "A BUNDLE VALUED RADON TRANSFORM, WITH APPLICATIONS TO INVARIANT WAVE EQUATIONS"
by: Thomas P. Branson, Gestur Olafsson and Henrik Schlichtkrull

222/92 On the Representations of some Infinite Dimensional Groups and Algebras Related to Quantum Physics
by: Johnny T. Ottesen

223/92 THE FUNCTIONAL DETERMINANT
by: Thomas P. Branson

224/92 UNIVERSAL AC CONDUCTIVITY OF NON-METALLIC SOLIDS AT LOW TEMPERATURES
by: Jeppe C. Dyre

225/92 "HATMODELLEN" Impedansspektroskopi i ultrarent en-krystallinsk silicium
af: Anja Boisen, Anders Gorm Larsen, Jesper Varmer, Johannes K. Nielsen, Kit R. Hansen, Peter Bøggild og Thomas Hougaard
Vejleder: Petr Viscor

226/92 "METHODS AND MODELS FOR ESTIMATING THE GLOBAL CIRCULATION OF SELECTED EMISSIONS FROM ENERGY CONVERSION"
by: Bent Sørensen

227/92 "Computersimulering og fysik"
af: Per M.Hansen, Steffen Holm, Peter Maibom, Mads K. Dall Petersen, Pernille Postgaard, Thomas B.Schrøder, Ivar P. Zeck
Vejleder: Peder Voetmann Christiansen

228/92 "Teknologi og historie"
Fire artikler af:
Mogens Niss, Jens Høyrup, Ib Thiersen, Hans Hedal

229/92 "Masser af information uden betydning"
En diskussion af informationsteorien i Tor Nørretranders' "Mærk Verden" og en skitse til et alternativ basseret på andenordens kybernetik og semiotik.
af: Søren Brier

230/92 "Vinklens tredeling - et klassisk problem"
et matematisk projekt af
Karen Birkelund, Bjørn Christensen
Vejleder: Johnny Ottesen

231A/92 "Elektrondiffusion i silicium - en matematisk model"
af: Jesper Voetmann, Karen Birkelund, Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B.Hansen

231B/92 "Elektrondiffusion i silicium - en matematisk model" Kildetekster
af: Jesper Voetmann, Karen Birkelund, Mette Olufsen, Ole Møller Nielsen
Vejledere: Johnny Ottesen, H.B.Hansen

232/92 "Undersøgelse om den simultane opdagelse af energiens bevarelse og isærdeles om de af Mayer, Colding, Joule og Helmholtz udførte arbejder"
af: L.Arleth, G.I.Dybkjær, M.T.Østergård
Vejleder: Dorthe Posselt

233/92 "The effect of age-dependent host mortality on the dynamics of an endemic disease and Instability in an SIR-model with age-dependent susceptibility
by: Viggo Andreasen

234/92 "THE FUNCTIONAL DETERMINANT OF A FOUR-DIMENSIONA BOUNDARY VALUE PROBLEM"
by: Thomas P. Branson and Peter B. Gilkey

235/92 OVERFLADESTRUKTUR OG POREUDVIKLING AF KOKS
- Modul 3 fysik projekt -
af: Thomas Jessen

236a/93 INTRODUKTION TIL KVANTE HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

236b/93 STRØMSSAMMENBRUD AF KVANTE HALL EFFEKTEN

af: Anja Boisen, Peter Bøggild

Vejleder: Peder Voetmann Christiansen
Erland Brun Hansen

237/93 The Wedderburn principal theorem and Shukla cohomology

af: Lars Kadison

238/93 SEMIOTIK OG SYSTEMEGENSKABER (2)

Vektorbånd og tensorer

af: Peder Voetmann Christiansen

239/93 Valgsystemer - Modelbygning og analyse

Matematik 2. modul

af: Charlotte Gjerrild, Jane Hansen, Maria Hermannsson, Allan Jørgensen, Ragna Clauson-Kaas, Poul Lützen

Vejleder: Mogens Niss

240/93 Patologiske eksempler. Om sære matematiske fisks betydning for den matematiske udvikling

af: Claus Dræby, Jørn Skov Hansen, Runa Ulsøe Johansen, Peter Meibom, Johannes Kristoffer Nielsen

Vejleder: Mogens Niss

241/93 FOTOVOLTAISK STATUSNOTAT 1

af: Bent Sørensen

242/93 Brovedligeholdelse - bevar mig vel

Analyse af Vejdirektoratets model for optimering af broreparationer

af: Linda Kyndlev, Kare Fundal, Kamma Tulinius, Ivar Zeck

Vejleder: Jesper Larsen

243/93 TANKEEKSPERIMENTER I FYSIKKEN

Et 1.modul fysikprojekt

af: Karen Birkelund, Stine Sofia Korremann

Vejleder: Dorthe Posselt

244/93 RADONTRANSFORMATIONEN oq dens anvendelse i CT-scanning

Projektrapport

af: Trine Andreasen, Tine Guldager Christiansen, Nina Skov Hansen og Christine Iversen

Vejledere: Gestur Olafsson og Jesper Larsen

245a+b /93 Time-Of-Flight målinger på krystallinske halvledere
Specialerapport

af: Linda Szkotak Jensen og Lise Odgaard Gade

Vejledere: Petr Viscor og Niels Boye Olsen

246/93 HVERDAGSVIDEN OG MATEMATIK - LÆREPROCESSER I SKOLEN

af: Lena Lindenskov, Statens Humanistiske Forskningsråd, RUC, IMFUFA

247/93 UNIVERSAL LOW TEMPERATURE AC CONDUCTIVITY OF MACROSCOPICALLY DISORDERED NON-METALS

by: Jeppe C. Dyre

248/93 DIRAC OPERATORS AND MANIFOLDS WITH BOUNDARY

by: B. Booss-Bavnbek, K.P.Wojciechowski

249/93 Perspectives on Teichmüller and the Jahresbericht Addendum to Schappacher, Scholz, et al.

by: B. Booss-Bavnbek

With comments by W.Abikoff, L.Ahlfors, J.Cerf, P.J.Davis, W.Fuchs, F.P.Gardiner, J.Jost, J.-P.Kahane, R.Lohan, L.Lorch, J.Radkau and T.Söderqvist

250/93 EULER OG BOLZANO - MATEMATISK ANALYSE SET I ET VIDENSKABSTEORETISK PERSPEKTIV

Projektrapport af: Anja Juul, Lone Michelsen, Tomas Højgård Jensen

Vejleder: Stig Andur Pedersen

251|93 Genotypic Proportions in Hybrid Zones

by: Freddy Bugge Christiansen, Viggo Andreasen and Ebbe Thue Poulsen

252|93 MODELLERING AF TILFÆLDIGE FÆNOMENER

Projektrapport af: Birthe Friis, Lisbeth Helmgaa Kristina Charlotte Jakobsen, Marina Mosbæk Johannessen, Lotte Ludvigsen, Mette Hass Nielsen

253/93 Kuglepakning

Teori og model

af: Lise Arleth, Kåre Fundal, Nils Kruse

Vejleder: Mogens Niss

254/93 Regressionsanalyse

Materiale til et statistikkursus

af: Jørgen Larsen

255/93 TID & BETINGET UAFHÆNGIGHED

af: Peter Harremoës

256/93 Determination of the Frequency Dependent Bulk Modulus of Liquids Using a Piezo-electric Spherical Shell (Preprint)

by: T. Christensen and N.B.Olsen

257/93 Modellering af dispersion i piezoelektriske keramikker

af: Pernille Postgaard, Jannik Rasmussen, Christina Specht, Mikko Østergård

Vejleder: Tage Christensen

258/93 Supplerende kursusmateriale til "Lineære strukturer fra algebra og analyse"

af: Mogens Brun Heefelt